Assessment of Developmental, Quantitative Literacy, and Precalculus Programs

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From an assessment perspective, developmental, quantitative literacy, and pre-calculus courses have many similarities and interrelationships. At many institutions, these courses constitute most of the department's workload. They are not generally the courses in which most faculty members invest their greatest enthusiasm or concern: that is usually reserved for courses for mathematics majors (or perhaps, in universities with graduate programs, for graduate students). They are the least mathematically interesting courses we teach. Moreover, since they're usually filled with students who dislike and fear mathematics and would rather be anywhere except in mathematics class, these are often the most difficult and frustrating courses to teach.

As a result, however, these are courses in which effective assessment can yield the greatest improvement in faculty working conditions as well as in student learning. If mathematics departments can turn these courses from ones students just muddle through into courses in which they grow in mathematical confidence and competence, these courses can become enjoyable and interesting to teach.

Colleges and universities are under pressure to develop assessment programs primarily of two types: for majors (often including other subjects required by a major), and for general education. The latter emphasis often leads to requests to mathematics departments to assess quantitative literacy. Pressures to assess developmental mathematics programs, on the other hand, typically reflect concerns about finances or about rates of student progress toward graduation. If students must repeat developmental courses several times before succeeding, or if they pass the developmental courses only to fail the credit-bearing courses for which they are prerequisites, students either graduate late or drop out entirely.

Interactions among these programs

There are no sharp boundaries between developmental, quantitative literacy, and precalculus courses. Some institutions require no mathematics; at some others, general education requirements are met by simply passing a placement exam or the developmental courses. Still others assume that adequate quantitative literacy skills will be developed through college algebra or precalculus courses. Often precalculus courses are directed at students planning to take calculus, but in fact are taken primarily to satisfy general education requirements. When this happens, faculty view the high DWF rate (D/Withdraw/Failure) in such courses as "casualties on the road to calculus." In reality, few of these students were ever on that road.

Programs assessing one of these components are likely to involve the others simply because of their interactions. The success of a developmental course is measured in large part by its students' success in their later quantitative literacy, college algebra, and precalculus courses. An assessment of precalculus courses needs to consider to what extent the courses are being used to meet needs for quantitative literacy, either by design or *de facto*, as well as how well they prepare students for calculus.

Since all of these courses are primarily for non-majors, good planning and good assessment should extend beyond the mathematics department. As with courses for students in mathematics-intensive majors, ensuring that the needs of other majors are met, as well as those of the institution, can do wonders for the reputation of the mathematics department on campus and can lead to support for its other initiatives (Chipman 1999).

The role of an effective placement process. In all of these courses, effective placement can be crucial to student success. It is very difficult to teach a class well when half the class has the prerequisites for the course and half does not. Many institutions, for example Arizona Western College (p. 47) and the University of Arizona (Krawczyk & Toubassi 1999), have found that replacing a voluntary placement process (where students may register for courses other than those they are placed into) by one in which students cannot take a course until they meet the prerequisites by placement or by taking courses makes a large difference in student success rates. This has also been my experience at Monmouth University.

Good placement processes generally involve multiple components (Cederberg 1999). A single examination cannot place students as accurately as can a process that combines this test result with information on high school rank, grade point average, last mathematics course taken, how long since that last course was taken, and SAT or ACT scores. As a project at Virginia Tech revealed, student self-descriptions in terms of how good they are at mathematics can also be an effective component of the placement procedure (Olin & Scruggs 1999).

Assessing the effectiveness of the placement process itself. Examining how well each factor and the overall formula predicts success and adjusting the formula in response to this analysis is an important part of the assessment of these introductory courses. National placement examinations such as Accuplacer[1] are appropriate only if the skills they test are those students need for the courses they are actually taking. For example, the skills needed for success in quantitative literacy courses are generally quite different from those needed for college algebra or calculus. Many locally-written placement tests also ignore this criterion.

The assessment cycle, applied to these clusters of courses

Examining goals and learning objectives. More than with most other courses that a mathematics department offers, there is often a substantial gap between course content and the course's role in the curriculum. Ideally, a discussion of goals will lead to changes in curriculum (often new courses) and improved student learning. When development of course goals begins with questions about desired student outcomes (e.g., "what do we want students to get out of this course? what should they be able to do when they've finished?"), it heads off the litany of faculty complaints about the students' lack of abilities or work ethic. The resulting goals will include statements of mathematical skills (e.g., "students should recognize when a linear model is appropriate to consider, be able to develop this model from given data, and make predictions from the model"), but usually also some broader skills (e.g., "students should be able to read critically a newspaper article involving graphs") and perhaps some affective outcomes (e.g., "students should feel less mathophobic at the end of the course"). Partner disciplines and committees on general education can provide useful input as mathematics departments define their course goals. Mount Mary College (p. 59) and Allegheny College (p. 37) discovered that simply detailing course goals for their developmental and quantitative literacy courses led to development of more appropriate courses.

From goals to objectives. To be able to assess goals effectively, they need to be made concrete. This is done by developing, for each goal, one or more learning objectives specifying skills students must develop to meet that goal. Concrete learning objectives can be developed even for affective goals. For example, a learning objective for the goal "students should feel less mathophobic ..." might be that "students will attempt to solve problems of types never before encountered, rather than skipping them entirely." Of course, it is easier to develop learning objectives for goals that are more specifically related to mathematical content, and there are likely to be more learning objectives for each of these goals. A report from King's College offers helpful discussion and examples of the difference between goals and learning objectives (Michael, 1999).

Sharing goals and objectives with all constituents. In larger institutions there are typically several sections of these

[1] www.collegeboard.com/highered/apr/accu/accu.html

introductory courses each semester, often taught by adjunct faculty or graduate students. In such cases, greater uniformity of student learning can be achieved by sharing with all faculty involved in teaching these courses—and perhaps also with the students—a clear statement of the goals and learning objectives of the courses, of how each of the objectives is to be achieved in the course, and how it will be assessed. Oakland University developed helpful information sheets for just this kind of purpose (Chipman 1999).

Choosing appropriate assessment mechanisms. Carefully chosen learning objectives often lead naturally and easily to appropriate assessment mechanisms. Timed, in-class tests are appropriate for assessing the particular mathematical content required in successor courses. However, other tools are often more effective for assessing affective or conceptual skill development. For example, if your objective is that students at least attempt to solve a problem, using a test question for which partial credit will be given is more likely to give this information, as are activities done under less time pressure and in less stressful situations than in-class examinations.

Sometimes it is effective to have mathematics' partner disciplines administer some of the assessments. For example, at the beginning of a psychology class that is going to use students' quantitative or algebraic skills, a brief quiz over prerequisite concepts can give both the psychology instructor and the mathematics department useful information on what has been retained. See the case study from Portland State University in this volume (p. 65) and in an earlier report from the University of Wisconsin and North Dakota State University (Martin & Bauman 1999) for good examples of using client departments to give this kind of feedback.

The studies at San Jose State University (p. 75)and Virginia Polytechnic Institute (Olin & Scruggs 1999) show that students' attitudes toward mathematics and how it is learned, and toward the courses themselves, can significantly affect their performance. While *surveys of student attitudes* cannot *alone* assess student learning, giving such surveys early in a course and working on improving students' beliefs about what they must do to succeed in mathematics can affect students' success. At Richard Stockton College of New Jersey (Ellen Clay 1999) and at Ball State University (Emert 1999) students write a class mission statement for quantitative literacy courses, while at St. Cloud State University students are asked to reflect on what they can do to improve their chances for success (Keith 1999). Students at St. Cloud State use journal articles in their planned major to explore the relevance of their study of mathematics to their future careers.

Developmental, quantitative literacy, and precalculus courses are particularly good places to use a range of *formative assessment techniques*. These are ways to find out what students do and don't understand about what has been presented, and at the same time to help students develop desired skills. For example, by having students write explanations of their answers to questions, the instructor learns what their confusions are, and the students have to think through what they understand. An example from the University of Southern Colorado (Barnett 1999) uses expanded true-false questions: "Determine if each of the following is true or false, and give a complete written argument for your response." An instructor at Surry Community College used "concept maps" to learn what students think they know about a topic (Atkins 1999). Many programs have students learn by writing about mathematics and explore ideas by working in groups. A lot of information on using this kind of formative assessment can be found in the section on "Assessment in the Individual Classroom" of *Assessment Practices in Undergraduate Mathematics* (Gold, *et al. 1999*).

Completing the cycle: using the data. In any type of assessment, the point of the exercise is to complete the cycle by using the data collected to improve learning. Usually the results of the first assessment activity raise more questions than they answer. If students aren't doing well in a follow-up course, what is the cause? Looking at the program, you can come up with some conjectures; these lead to further assessment activities. Once you find the causes, it's time to look at how to *change the program*. Since almost all of the programs considered here involve partner disciplines or university committees, revision should include not only the mathematics department but these other constituencies as well. This takes more time but yields many benefits for the department due to the good will gained. Finally, of course, you need to start the cycle again: as you redesign courses, consider goals, rethink learning objectives, and decide how to assess the effectiveness of the changes.

Developmental courses

The goal for developmental courses seems clear: prepare students for credit-bearing courses. However, judging by the course content, the goal often *appears* to be remediating what students haven't learned in grades K–12. These two goals may be quite different. High schools attempt to prepare children for all possible educational futures, including majoring in mathematics. Once the student is in college, that educational aim may be much better defined, and the student may not ever need to study calculus. So remediating the high school deficiencies may not be necesssary or appropriate. Sometimes, however, the institution's view may be that every student should master certain mathematical skills. If some of these are normally mastered at the pre-college level, achieving these skills does become one of the goals of the developmental program. Depending on the college's view of its general education expectations, the developmental courses may be used to meet general education needs as well (in which case, as goals are developed, that expectation must be included). It is also not uncommon for developmental courses to be the prerequisites for various courses in the sciences and social sciences, in which case these partner disciplines should be invited to communicate their expectations.

Tracking student success rates. Assuming that the primary purpose of developmental courses is to prepare students for further courses in mathematics, the most direct way to assess this objective is to investigate student success in these later courses. For an overall, thumbs-up/thumbs-down, answer, this involves tracking these students in these later courses. Several case studies report on this kind of effort: Allegheny College (p. 37); Cloud County Community College (p. 55); and earlier, St. Peter's College (Poiani,1999).

There are two obvious approaches to this investigation. One option is to get a list of all students in the follow-on courses and their grades, look to see which of these students took developmental courses, and compare that group's grades with students overall. The other approach is to look in the other direction: follow students from the developmental courses to see how they do in their later mathematics courses.

If you find as did Allegheny College (p. 37) that students who ignore placement into developmental courses and enroll directly in the credit-bearing courses do better than students who take and pass the developmental courses first, you clearly need to investigate further. It may be the placement process that needs revision, not the developmental offerings. Perhaps a factor such as student self-reporting on how hard they work or their mathematical confidence needs to be included. Or the developmental students may wait a long time before taking the credit-bearing course. You can control for this by looking at the time elapsed between taking the developmental course and its successor. But it also may be that the developmental courses really need serious restructuring.

Generally in assessment, the answer to one question leads to another question, or to making the question more precise. Effectiveness of a developmental program can be further investigated by giving pre-tests at the beginning of the credit-bearing courses to determine whether students, by the time they take the successor course, have the skills needed for success in that course. You may need to add some questions beyond computational multiple-choice problems: problem-solving questions, open-ended questions where you look at the methods students use, for example. This is a finer sieve than simply tracking student success rates, as you can determine precisely which skills students are either not learning or not retaining. This information can then be translated directly into changes of emphasis or teaching methods in the developmental courses, and perhaps (when it's an issue of non-retention due to a large time lapse between the courses) changes in student advising. (The report from Arizona Western College (p. 47) offers a good example of this.)

Attitude surveys. A second (and generally secondary) method sometimes used for assessing developmental courses is attitude surveys. Often these are used more for formative than summative assessment, but learning that students' attitudes about their mathematical abilities or the value of mathematics has not changed can be a red flag warning that the course isn't achieving its goals. The case study from Mount Mary College (p. 59) discusses such a survey. Surveys of faculty in subjects that have the developmental courses as their mathematical prerequisites, asking what mathematical weaknesses they feel their students have, can also indicate areas which need improvement.

Formative assessment. Developmental students are perhaps the group that benefits most from the use of a variety of formative assessment methods such as using group work or brief writing assignments during class time. These make the course less dry, force students to reflect on the computations they're learning, and take the course beyond simply repeating unsuccessful high school experiences at a faster pace.

Quantitative literacy courses

Quantitative literacy courses are mathematics' most amorphous courses. There is an enormous variety of mathematics that can inform the thinking of an educated citizenry, but this cannot all be crammed into the one or two courses required of students to meet general education requirements.

In 1996, the Quantitative Literacy Subcommittee of the Committee on the Undergraduate Program in Mathematics issued guidelines for quantitative literacy programs (Sons, 1996). An excellent description of quantitative literacy and a summary of the CUPM recommendations appeared in *Assessment Practices in Undergraduate Mathematics* (Sons, 1999). These reports argue that a college graduate should be able to:

- interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them;
- represent mathematical information symbolically, visually, numerically, and verbally;
- use arithmetical, algebraic, geometric and statistical methods to solve problems;
- estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results;
- recognize that mathematical and statistical methods have limits.

Other expressions of goals for quantitative literacy (Dwyer, to appear; Steen 1997; Steen 2001) convey different ranges of expectations for example:

- estimating answers and checking answers for reasonableness;
- understanding the meaning of numbers;
- using common sense in employing numbers as a measure of things.

The primary recommendation of the CUPM report is that quantitative literacy cannot be achieved in a single course, but should involve at least two courses, a foundational experience (often, but not necessarily, taught in the mathematics department) followed by a continuation experience, often within the student's major. An alternative to the two-course model for building a rich quantitative literacy experience at the college level is to infuse quantitative literacy across the general education program. It is important to get faculty members from a full range of disciplines involved in the development of courses to meet this recommendation, and a survey of faculty on what they see as the quantitative literacy needs of their students can be a good starting point.

Recognition that we cannot do everything in one course gives us the freedom to look at the set of skills and understandings we want students to develop in this kind of course, and find multiple ways of meeting these goals. This often leads to a great variety of content in these courses, even within the same institution. To assess how well these courses meet students' quantitative literacy needs, we must look not only at students' success in learning the mathematical content of a given course.

A well-designed examination may well be a good beginning, as it can be used as a pre- and post-test across a range of quantitative literacy courses, and even in the continuation experience courses. If course instructors are given the results of pretests, they can find a level of course presentation more appropriate to the abilities the students bring to the course. King's College (Michael 1999) and Virginia Commonwealth University (pp. XX–YY) have used some kind of pre/post test for such courses; the Portland State psychology department (pp. XX–YY) offers an example of such testing in "continuation experience" courses.

However, many quantitative literacy goals cannot be assessed solely via a multiple choice examination. There should be at least a free-response portion to assess students' ability to represent mathematical information and to test their ability to interpret the results of calculations. A common pitfall is to write the test questions without correlating them with the program goals. It is also difficult to write questions that are not course-content specific and thus can be used to compare students' development across a range of such courses. Sons' article (1999) discusses issues such as being able to compare student learning across courses that may have very different content.

Many alternative assessment methods are particularly useful in quantitative literacy courses, both formatively and summatively: portfolios, journals, using writing to learn mathematics, having students create problems for use either as review for an exam or as exam questions themselves, projects, group work (Gold, et al., 1999, especially part II). To use these alternative methods for summative assessment of the quantitative literacy program, rubrics must be developed to enable readers to summarize rapidly masses of information in ways that give useful information that can be compared across courses. Sometimes this can be done as faculty members grade the activity in the course (but separately from the grade for the activity) if the rubrics have been developed and faculty members trained in their use in advance.

College algebra and precalculus courses

When most go on to calculus. The biggest assessment challenge in college algebra and precalculus is deciding what the goals are. As long as the courses are primarily being used to develop skills necessary for students to succeed in calculus, the goals are fairly clear, and the assessment issues are similar to those for developmental courses. It is, however, important to determine whether this is in fact their primary use. If it is, initial assessment methods can involve looking at how well students succeed in the next course, viz. calculus. The case study from San Jose State University (p. XX–YY) illustrates just this situation.

A more detailed analysis requires study of the particular skills students need in calculus and how well they do on those skills, both on final exam questions in the precalculus course and on similar questions given at the beginning of the calculus course. Often items from the department's placement test can be the initial questions used in such an assessment. It is important, however, to correlate students' scores on these questions at the beginning of calculus with their grades at the end to make sure that these questions really do test skills necessary for success in calculus.

However, multiple choice placement test items cannot test all the skills students will need for success in calculus. Students need to be able to translate word problems into algebraic equations, to translate between multiple representations of data and functions, and much more. Often examining student work on one long-answer question can pinpoint the range of confusions that are resulting in incorrect answers on multiple-choice questions.

Surveys can be used in these courses, as in developmental courses, to examine student attitudes towards mathematics and how it is learned. Often students who had calculus in high school place into precalculus in college not because of lack of ability but because they haven't yet learned how to study mathematics. At San Jose State (p. XXX-YYY) successful students showed a significant understanding of what was needed to succeed in these courses.

When few go on to the calculus sequence. On the other hand, if you find as did Allegheny College (p. XX–YY) that the majority of students in college algebra/precalculus are not going on to a full year of calculus, you may want to consider restructuring the courses. Often courses called college algebra or precalculus are used to meet at least three different needs: they serve as the quantitative literacy (and thus, terminal mathematics) course for a large number of students; they prepare students in business and biology for an applied calculus course; and they also try to prepare students to succeed in the mainstream calculus sequence. They are trying to meet the needs of three very different audiences with extremely different goals. I've yet to see a program that recognizes and acknowledges all these goals and successfully meets them in a single course. Discussions with faculty in partner disciplines can help mathematics departments decide what combination of courses will be most effective in their particular context. If after such discussions the pre-calculus or college algebra course is still left meeting the needs of partner disciplines (in addition to preparing students for mainstream calculus) or helping students develop quantitative reasoning skills, the assessment program for these courses must include activities that assess these other goals in addition to the activities mentioned here that assess preparation for calculus.

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